

Modeling Stochastic Eye Tracking Data: A Comparison of Quantum Generative Adversarial Networks and Markov Models

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Research objectives



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- Assess the effectiveness of Quantum Generative Adversarial Networks (QGANs) in modeling eye movement velocity data.

[a] *Lencastre et. al., 2023. Modern AI versus century-old mathematical models: How far can we go with generative adversarial networks to reproduce stochastic processes? Physica D: Nonlinear Phenomena 453 (2023)*

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- To evaluate the performance of QGANs in reproducing stochastic trajectories and compare them to classical GANs [a] and Markov models.

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- Assess the effectiveness of Quantum Generative Adversarial Networks (QGANs) in modeling eye movement velocity data.
- To evaluate the performance of QGANs in reproducing stochastic trajectories and compare them to classical GANs [a] and Markov models.
- Identify challenges and opportunities for optimizing QGANs to better align with real-world data characteristics.

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Introduction



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- The generator generates realistic data, while the discriminator differentiates between real and generated data.
- The process is **generative modeling**

Challenges of GANs



- Challendes of GAN:
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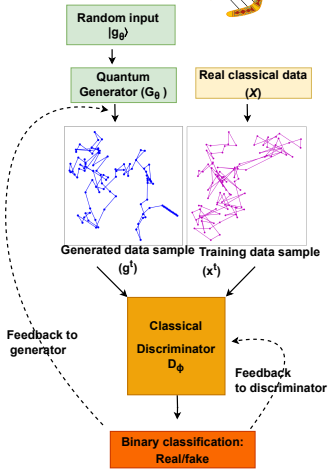
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- Challendes of GAN:
  - Difficulties in achieving stable training.
  - The vanishing gradient problem.
  - Mode collapse.
- Efforts from both the academic and commercial sectors have been made to address these problems.
- Quantum Deep Learning models are gaining popularity, motivated by the potential for quantum advantage in machine learning.

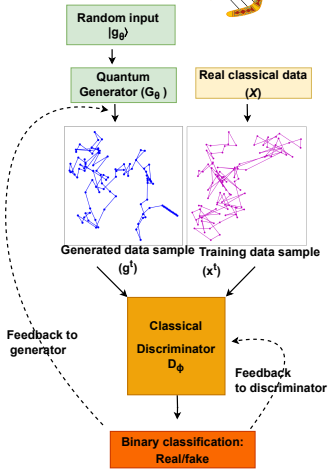
Quantum GAN model

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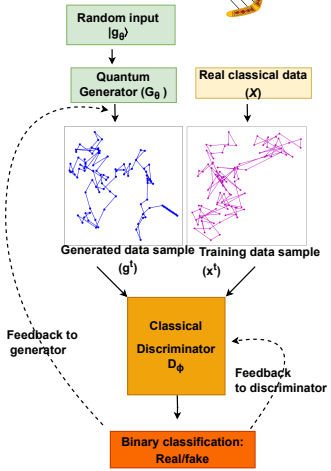
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- The discriminator is a classical feed-forward neural network implemented using PyTorch.



Quantum generator

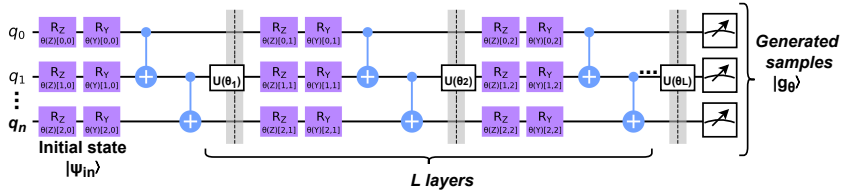


N-qubit input states to N-qubit output states

$$
\begin{aligned}
|g_\theta\rangle &= G_\theta \, |\psi_{in}\rangle \\
&= \prod_{p=1}^{2^N} \left(\bigotimes_{q=1}^{N} (R_Y R_Z(\theta^{q,p})) U(\theta_L) \right) \bigotimes_{q=1}^{N} (R_Y R_Z(\theta^{q,0}) \, |\psi_{in}\rangle) \\
&= \sum_{j=0}^{2^N-1} \sqrt{p_j} \, |j\rangle
\end{aligned}
$$

where samples are drawn by measuring the output state $|g_\theta\rangle$ in computational basis with measurement outcomes $|j\rangle$, $j \in \{0, 1...., 2^N - 1\}$.

Quantum generator



Quantum generator circuit in variational form with L layers acting on n qubits.

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- It comprises a three-layer LSTM network with 128 hidden units and bidirectional processing.
- Dropout is applied to prevent overfitting, and the output is a probability scalar representing the authenticity of the data.

Loss functions for GANs



Generator Loss

$$L_G(\phi, \theta) = -\frac{1}{m} \sum_{i=1}^{m} [\log D_\phi(g^l)]$$

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Discriminator Loss

$$L_D(\phi, \theta) = \frac{1}{m}\sum_{l=1}^{m}[logD_\phi(x^l) + log(1 - D_\phi(g^l))]$$

Markov-chain models of time-series



Markov Property:

$$Pr(X_{t=j} = \hat{x}_j | X_{t=j-1} = \hat{x}_{j-1}, \ldots, X_{t=0} = \hat{x}_0) = Pr(X_{t=j} = \hat{x}_j | X_{t=j-1} = \hat{x}_{j-1}), \quad (1)$$

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- This process relies on the robustness of the Markov assumption.
- The size of the state space and sample size influence the efficacy and computational efficiency of the model.

Data and statistical measures



Data Collection:

- Eye-tracking data collected at OsloMet using the Eye-link Duo device.
- Data recorded at 200 Hz, preprocessed to convert distances into velocities.
- Data normalized and structured into sequences of 100 data points for training.

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QGAN Training:

- Velocity data from eye movements used to train the discriminator.
- The discriminator learns to distinguish real data from synthetic data generated by the quantum generator.

Data and statistical measures



Jensen-Shannon Divergence:

$$D_{JS}(P\|Q) = \frac{1}{2}D_{KL}(P\|M) + \frac{1}{2}D_{KL}(Q\|M), \tag{2}$$

where $M = \frac{1}{2}(P + Q)$.

Data and statistical measures



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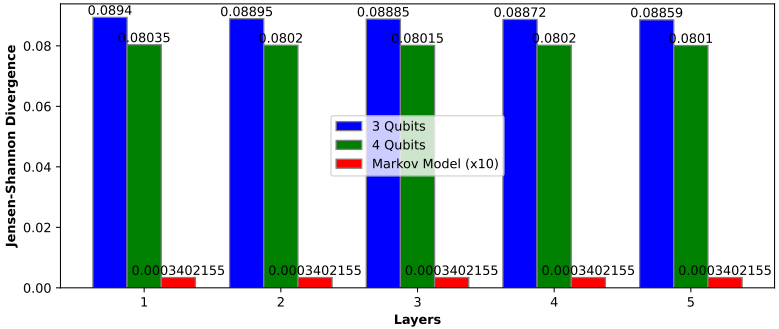
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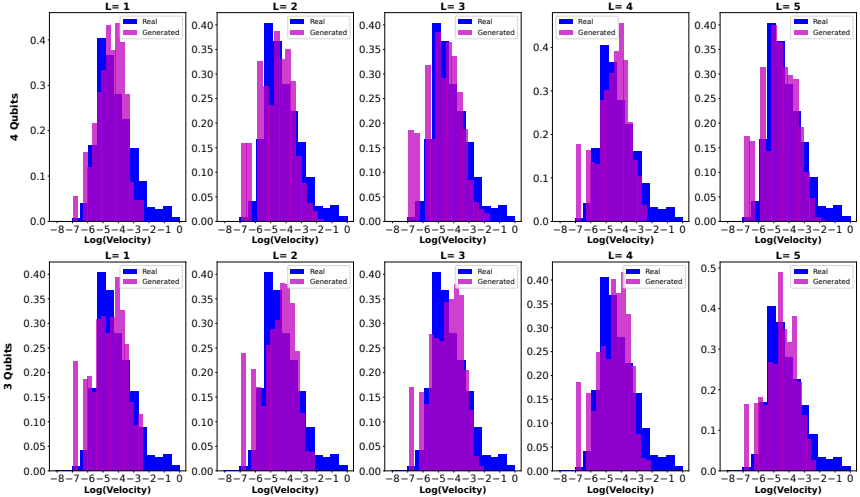
- JS divergence provides a symmetric measure between probability distributions.
- It is used to assess the performance of both GAN and Markov models in producing data that resemble real data distributions.

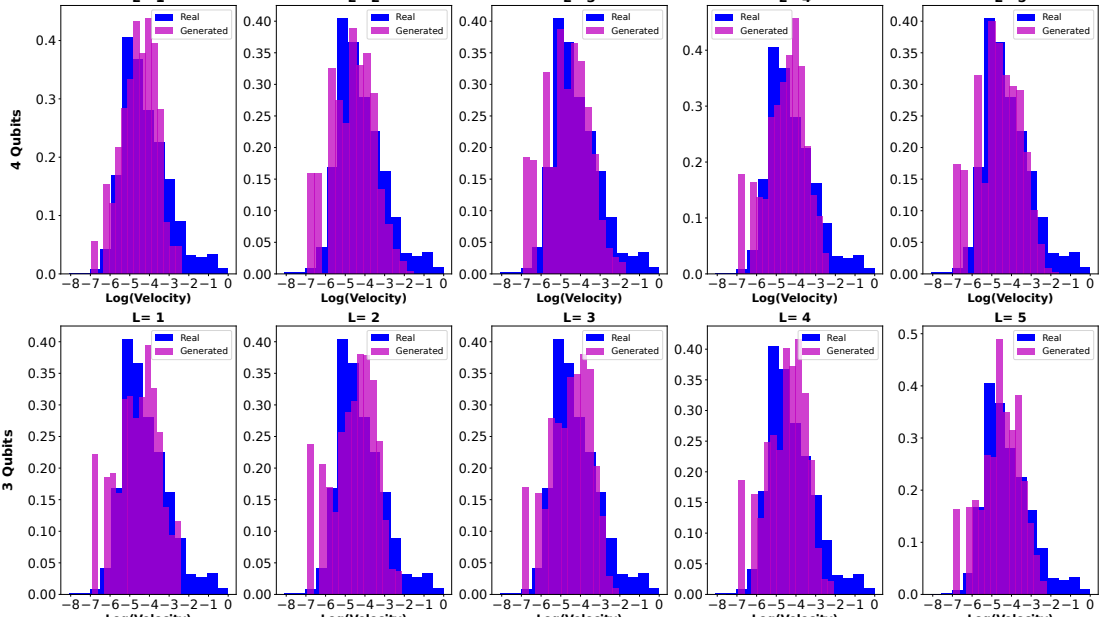
Statistical analysis

- QGAN models show increased skewness and kurtosis with deeper layers.
- Markov models consistently exhibit lower JSD, indicating a closer approximation to real data.

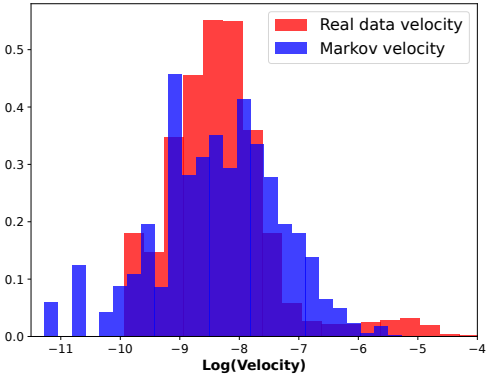


Results: QGAN




4 Qubits
3 Qubits
L= 1
L= 2
L= 3
L= 4
L= 5
Real
Generated
Log(Velocity)

Results: Markov model of time-series



Comparative histograms of log-transformed eye movement velocities: real versus Markov model.

Discussion and conclusions



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- Markov models outperform QGANs regarding data fidelity, highlighting the need for further advancements in quantum computing and algorithm development.
- Future work should focus on improving quantum algorithms and comparing quantum GANs with quantum Markov chains, and quantum Markov chains with classical Markov chains.



Thank you for your attention



